Physics Factsheet

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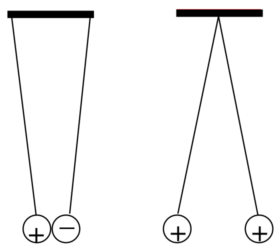
Coulomb's Law

You probably already know that electrically charged particles can exert a force on each other even when they are not actually touching. You may have carried out experiments with electrostatics which have shown you that:
- there are two kinds of electric charge – called positive (+) and negative (-);
- opposite charges (+ and -) attract one another;
- like charges (+ and +) or (- and -) repel one another.



Physicist Charles Coulomb came up with a law which described not only the direction, but also the size, of this force between charged particles. It is very similar in form to Newton's law of gravitation, but of course, gravitational forces are only ever in one direction – always attractive, never repulsive.

> ***Coulomb's law*** *states that the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.*
>
> $F = k\frac{Q_1Q_2}{r^2}$ *where* Q_1 *and* Q_2 *are the two charges (in* ***coulombs****)*
> *r is the distance (in* ***metres****) between them.*
> *F is the force in* ***newtons***
> *k is the constant of proportionality, and it has an equation of its own:*
>
> $k = \frac{1}{4\pi\varepsilon_0}$ *where* ε_0 *is called the permittivity of free space.*

The force between the two charged particles depends on what is between them. If anything other than empty space (a vacuum) comes between them, the force between the charges is reduced (which means that the permittivity is increased).

The permittivity of air at standard temperature and pressure is $1.0005 \times \varepsilon_0$ and so we can usually take ε_0 as the value for air as well as for free space. The value of the constant $\varepsilon_0 = 8.85 \times 10^{-12}$ N m^2 C^{-2} or Fm^{-1} (**farads** per metre)

So, we can combine all of this into the original mathematical form of Coulomb's equation:

> **Coulomb's equation:** $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
>
> You need to remember this equation and be able to use it to answer questions. (You won't have to remember the value of ε_0 - it will be given you in any exam.)

An approximate numerical value for $\frac{1}{4\pi\varepsilon_0}$ is 9×10^9,

so a useful form of Coulomb's law for making *rough* calculations is:

$$F = 9 \times 10^9 \frac{Q_1Q_2}{r^2}$$

If you have already done any work on Newton's law of gravitation, you will probably see the similarities. The force in each case depends on both of the objects causing it (charges for Coulomb, masses for Newton) and in each case it is an *inverse square* law.

This means for example that if the distance is doubled, the force gets four times smaller (because the inverse of the square of 2 is ¼ or, mathematically: $\frac{1}{2^2} = 0.25$)

> **Typical Exam Question**
> **(a) Calculate the force between two point charges of +100μC placed 125cm apart in a vacuum. ($\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1})**
> **(b) State what difference it would make to the force calculated if:**
> **(i) one of the charges was negative.**
> **(ii) the space between the charges was occupied by air.**
> **(iii) the space was occupied by paraffin (which has high permittivity)**
>
> *(a)* $F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r^2}$
>
> $= \frac{(100\times 10^{-6})(100\times 10\,10^{-6})}{4\pi(8.85\times 10^{-12})(1.25)^2} = 57.5\ N$ ✓
>
> *(b) (i) the force would be attractive rather than repulsive* ✓
>
> *(ii) there would be a very slight reduction in the size of the force (or, no change)* ✓
> *(because the permittivity of air is only very slightly higher than that of free space).*
>
> *(iii) there would be a large reduction in the force (because the permittivity of paraffin is many times higher than that of free space).* ✓

> ***Exam Hint****: Be very careful with the units and the powers of 10 in this sort of calculation. It is most sensible to convert everything to SI units at the beginning (metres, kilograms, seconds, amps etc) and work with them; for example, put 100μC and 125 cm into your equation as 100×10*$^{-6}$ *C and 1.25 m at the very beginning. Also, be very careful about how you enter these values into a scientific calculator – make sure you know exactly how to enter powers of 10. It is especially important in this sort of question where you don't usually have a 'common-sense feel' for what a reasonable final answer ought to be.*

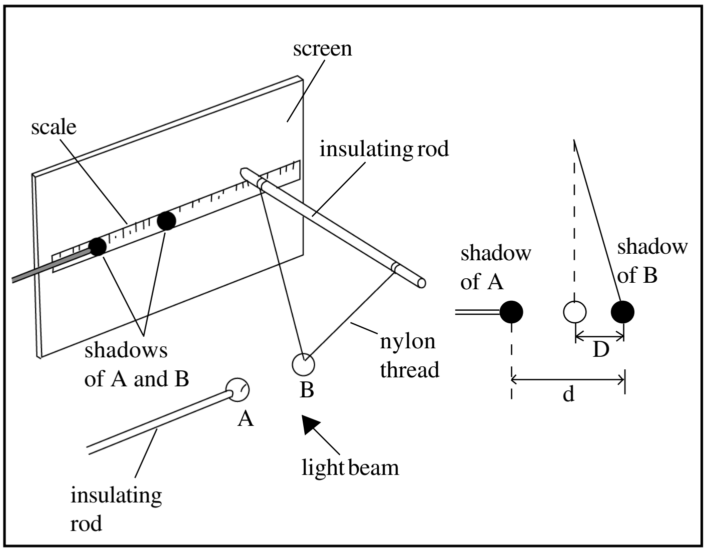
Practical considerations

A considerable proportion of the matter which makes up everything in the universe consists of charged particles and a knowledge of the forces between these particles is needed if we are to understand the structure of matter and the structure of the atom. The law as we have stated it applies to *point charges*, that is, charges which are concentrated in a single point. Sub-atomic particles – the protons and electrons within an atom – are so tiny that to all intents and purposes we can regard them as point charges. In fact, any uniformly charged conducting sphere (with a total charge Q) behaves – as far as external effects are concerned – as if it were a single point charge Q concentrated at the centre of the sphere. This only holds true if there are no other charges nearby to disturb the distribution of charge on the surface of the sphere. When these conditions are met, then Coulomb's law gives an acceptable approximation of the forces experienced by such a sphere and of the forces exerted by it on other charged particles.

Any experimental attempt to investigate Coulomb's law by measuring the forces between charged spheres is fraught with practical difficulties. The amount of charge we can investigate is difficult to measure with accuracy, the forces involved are small and the charge has a tendency to leak away into the air.

A popular method involves light spheres (usually polystyrene balls) coated with a conducting material such as graphite paint, one mounted on an insulating **perspex** handle and the other suspended from insulating nylon thread. The diagrams below show a suitable arrangement. The balls may be charged by touching each in turn with, for example, the plate of an electrophorus or some other electrostatic charging device (Fig1).

Fig 1. Coulomb's law apparatus



One of the charged balls is glued to the bottom of a 'V'-shaped swing of nylon thread so that it can only swing backwards or forwards in one direction. As the second charged ball, mounted on the insulating rod, is moved closer to the swinging ball, the amount of displacement can be measured by shining a beam of light from the front and marking the position of the shadows of both balls on a screen behind the apparatus. The deflection is proportional to the force causing it. If a few values of d (distance between the two balls) and D (deflection of the swinging ball from its start position) are measured, then it can be shown that $F = \frac{1}{r^2}$

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

a) State Coulomb's law for the force between two point charges [2]

$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ ✓ ✗

- One of the two marks available is awarded for explaining what the symbols (Q, r and ε_0) stand for. Easy to do, easy to forget to do!

b) Calculate the force on an electron placed at a point 3.3×10^{-13}m from a spherical nucleus whose charge is 1.12×10^{-18}C. (Take the charge on an electron (e) to be -1.6×10^{-19}C and $\varepsilon_0 = 8.85 \times 10^{-12}$ Fm^{-1}) [3]

Field strength = 1.31×10^{-13} ✗ ✗ ✗

- Disaster! The wrong answer. However – there are *three* marks available for this part of the question, and only one of them is for the correct final answer. This student has thrown away all three marks by not remembering the golden rule of all physics exams – **show your working!**
- The *only* error the student has made is to forget 'ε_0' when calculating the answer – an easy sort of mistake to make, especially when concentrating on entering all the powers of 10 correctly into the calculator. If the working had been shown, this would have still allowed two of the three marks for using the correct equation and making the substitution of numbers for symbols correctly.
- Final error – no units for the answer. Again, easy to forget at the end of a complex calculation – but the units are just as important a part of the answer as the numerical value.

c) Explain how an atom could be ionised by the application of a suitable external electric field [1]

An atom can be ionised by the application of a suitable external electric field because when something is ionised it means making it into ions (charged particles) and the electric field can do this. ✗

- It is very common for students to rewrite the question when they don't know the answer. This student has remembered something about ions from GCSE (that they are charged particles) but it doesn't make up to an adequate answer at this level – the fact remembered is not sufficient for this question. Repeating the same thing twice doesn't show any greater understanding.

Examiner's answers

a) *Coulomb's Law:* $F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ ✓

where: Q_1 & Q_2 are the sizes of the charges
r the distance apart
ε_0 the permittivity of the medium separating them ✓

b) $F = Q_1Q_2/4\pi\varepsilon_0 r^2$ ✓
$= (1.12 \times 10^{-18}) \times (-1.6 \times 10^{-19}) / (4\pi\ 8.85 \times 10^{-12}\ (3.3 \times 10^{-13})^2)$ ✓
$= -0.0148$ N ✓

c) The force on the electron due to the external field must be greater than the attractive force between the electron and the nucleus for ionisation to occur. ✓

Questions

1. A potential difference of 5.5 kV is applied to the electron gun of a cathode ray tube, in order to accelerate the electrons from rest. The charge on an electron is -1.6×10^{-19} C, and the mass of an electron is 9.1×10^{-31} kg. Calculate:
   (i) the kinetic energy of the electrons as they leave the electron gun. [2]
   (ii) the speed of the electrons [2]

2. (a) The arrow below illustrates a beam of electrons leaving the electron gun of a cathode ray tube. Complete the diagram to show what you would need to add to make the beam deflect downward using a d.c. supply. Draw and label the path of the beam. [3]

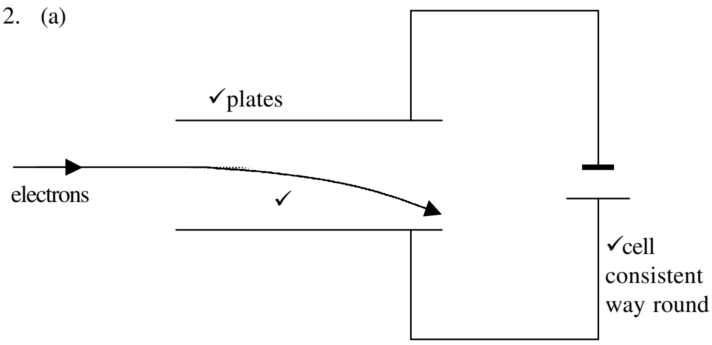
   (b) What difference would increasing the accelerating voltage make? Explain your answer. [2]

Answers

1. (i) E_k, the kinetic energy lost by each electron equals E_p, the potential energy lost as it falls through a voltage, V. ✓
   $= eV = 1.6 \times 10^{-19} \times 5.5 \times 10^{3} = 8.8 \times 10^{-16}$ J ✓ [2]

   (ii) $E_k = ½ mv^2$ ✓
   $v = (2E/m)^{1/2}$
   $= [(2 \times 8.8 \times 10^{-16})/(9.1 \times 10^{-31})]^{1/2}$ ✓ $= 4.4 \times 10^{7}$ ms^{-1} ✓ [2]

2. (a)



[3]

   (b) If the accelerating voltage is increased the electrons would leave the electron gun at a greater velocity than before. ✓ [1]

Acknowledgements:
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