Questions

First: Complete the following :

(1)

a) In Δ ABC if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called ………

b) The medians of the triangle intersect at ………………

c) The point of intersection of the medians of the triangle divides each of them in the ratio of ……… : ……… from the base.

d) The points which divides the median of the triangle in the ratio 1 : 2 from the base is the point of ………………

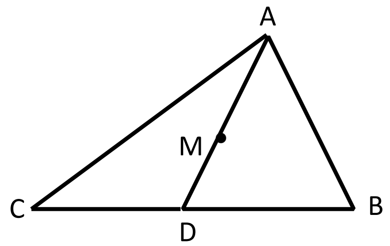
e) In the opposite figure:

If M is the point of intersection of the medians of Δ ABC then:

First: BD = ……………BC

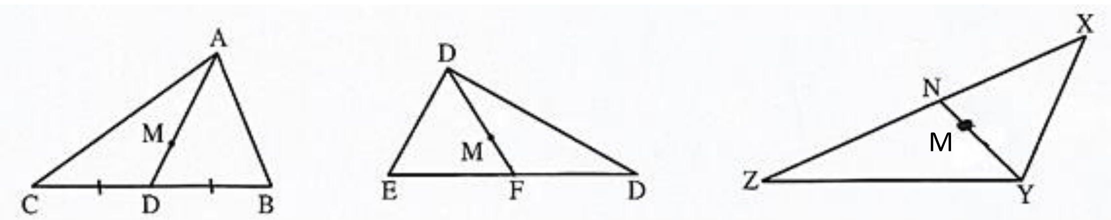
Second: AM = ……………… MD

Third: AM = ……………… AD



(2) In each of the following figures

M is the point of intersection of the medians of the given triangle.

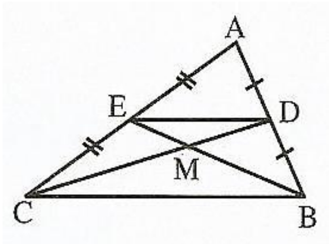


a) Fig. (1): If AM = 2 cm, then MD = ………… cm.

b) Fig. (2): If MF = 1.5 cm, then DF = ………… cm

c) Fig. (3): If YN = 6 cm, then YM = ………… cm

(3) In the opposite figure:



a) If : DE = 3 cm, then BC = ………….. cm.

b) If : CD = 4.5 cm, then CM = ………….. cm.

c) If : ME = 1.2 cm, then BE = ………….. cm.

(4)

a) The length of the median of the right angled triangle which is drawn from the vertex of the right angle equals …………..

b) If the length of the median of the triangle which is drawn from one of its vertices equal half the length of the opposite side to this vertex, then …………..

c) The length of the side opposite to the angle measure 30° in the right angled triangle equals …………..

(5) In each of the following figures:

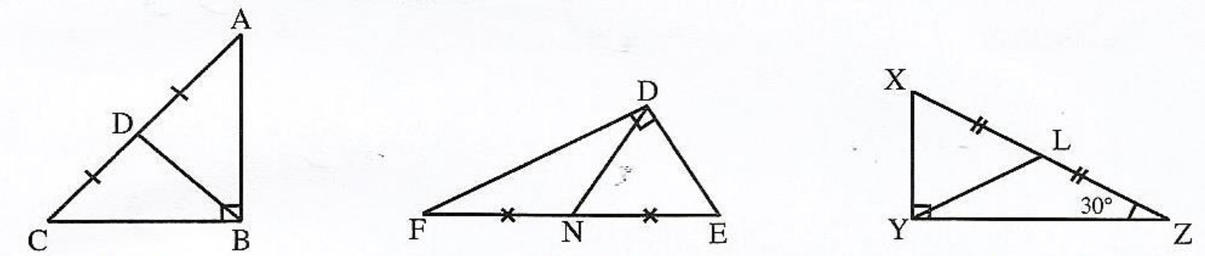


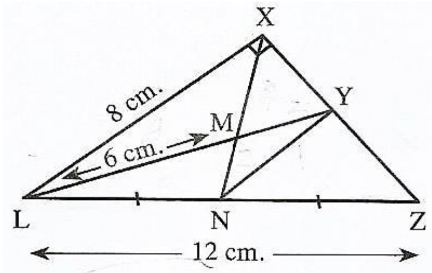
Fig. (1) Fig. (2) Fig. (3)

a) In fig. (1): If AC = 8 cm, then BD = ………….. cm.

b) In fig. (2): If DN = 3 cm, then EN = ………….. cm.

c) In fig. (3): If XY = 3.5 cm, then YL = ………….. cm.

(6) In the opposite figure:



$\overline{XN}$ and $\overline{YL}$ are two medians

m ($\angle$ ZXL) = 90°, ZL = 12 cm

XL = 8 cm, ML = 6 cm

a) XN = ………….. cm b) YN = ………….. cm

c) MY = ………….. cm d) YL ………….. cm

(7)

a) The base angles of the isosceles triangle are …………..

b) The measure of any angle of the equilateral triangle equals …………..

c) If two angles in a triangle are congruent then the two sides opposite to these two angles are …………..

d) If the angles of any triangle are equal in measure then ………

e) If the measure of an angle in the isosceles triangle is 60° then the triangle is …………..

f) If Δ ABC is an equilateral triangle then m ($\angle$ B) = …………..°

(8)

a) If XYZ is a right angled triangle at Y and XY = YZ then m ($\angle$ X) = …………..

b) ABC is an isosceles triangle where AB = AC and m ($\angle$ A) = 110°, then m ($\angle$ B) = …………..

c) ABC is an isosceles triangle and the measure of one of the two base angles equals 65° then the measure of the vertex angle in this triangle equals …………..

d) XYZ is an isosceles triangle where XY = XZ if m (∠ X) = 80°

then m (∠ Y) = …………..

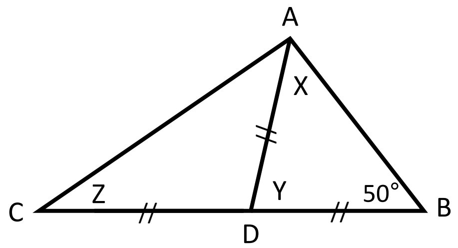
e) In Δ ABC if $\overline{AB} \perp \overline{BC}$ and AB = BC then m (∠ A) = …………..

(9) In the opposite figure:

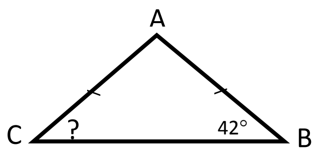
a) X = …………..

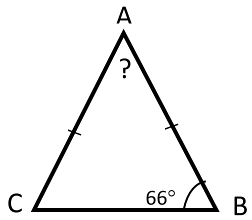
b) Y = …………..

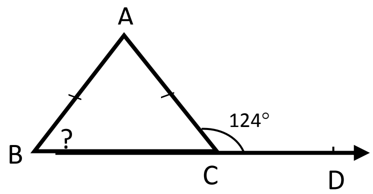
c) Z = …………..



(10) Complete using data registered on each figure:







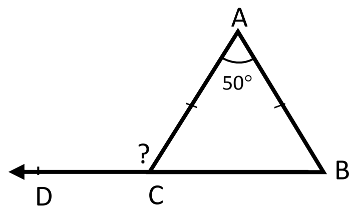


Fig. (1) m (∠ C) =…………..

Fig. (2) m (∠ A) = …………..

Fig. (3) m (∠ B) = …………..

Fig. (4) m (∠ D) = …………..

Second: Choose the correct answer from those given:

1. If M is the point of intersection of the medians of Δ ABC and D is the midpoint of $\overline{BC}$, then AD = …………..

a) 2 Am b) $\frac{2}{3}$ MD c) $\frac{3}{2}$ AM d) 4 MD

2. The point of intersection of the medians of the triangle divides each of them with the ratio ………….. from the vertex.

a) 2 : 1 b) 1 : 2 c) 3 : 1 d) 3 : 2

3. If M is the point of intersections of the medians of the triangle in Δ ABC and $\overline{AX}$ is a median of length 6 cm, then AM equals …………

   a) 1 b) 2 cm c) 3 cm d) 4 cm

4. ABCD is a rectangle M is the point of intersection of its diagonals. If the length of the diagonal is 6 cm, then the length of the median $\overline{AM}$ equals …………..

   a) 2 cm b) 3 cm c) 6 cm d) 12 cm

5. The measure of the exterior angle of the equilateral triangle equals …………..

   a) 30° b) 60° c) 90° d) 120°

6. If the measure of the vertex angle of the isosceles triangle equals 50°, then the measure of each angle of its base equal …………..

   a) 40° b) 65° c) 70° d) 130°

7. If the measure of one of the two base angles of the isosceles triangle equals 40°, then the measure of the vertex angle is …………..

   a) 40° b) 50° c) 80° d) 100°

8. The base angles of the isosceles triangle are …………..

   a) complementary b) supplementary
   c) congruent d) straight angles

9. If XA = XB and YA = YB then $\overleftrightarrow{XY}$ ………….. $\overline{AB}$

   a) // b) ⊥ c) = d) ≡

10. If A lies on the axis of symmetry of $\overline{XY}$ then $\overline{AX}$ ………….. $\overline{AY}$

   a) // b) ⊥ c) = d) ≡

11. The quadrilateral ABCD in which $\overleftrightarrow{BD}$ is an axis of symmetry of $\overline{AC}$ may be …………..

   a) a rhombus b) a rectangle
   c) a parallelogram d) a trapezium

12. If AX = AY and BX = BY where X and Y are at different sides of $\overline{AB}$ then $\overleftrightarrow{XY}$ ………….. $\overline{AB}$

a) //   b) ⊥   c) =   d) ≡

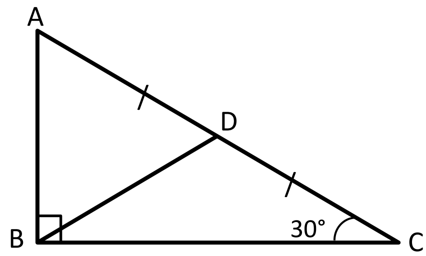
Third: Questions for getting the answer:

(1) In the opposite figure:

m (∠ ABC) = 90° , D is the midpoint of $\overline{AC}$,

m (∠ C) = 30°

Prove that: Δ ABD is equilateral



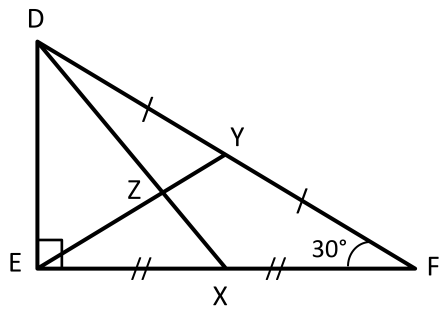
(2) In the opposite figure:

m (∠ DEF) = 90° ,

X and Y are the midpoints of $\overline{EF}$, $\overline{DF}$

respectively, m (∠ F) = 30°

DF = 12, XZ = 2.5 find the perimeter of Δ DEZ



(3) In the opposite figure:

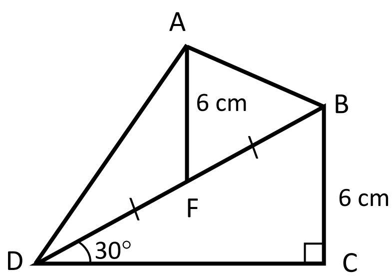
m (∠ C) = 90°, $\overline{AF}$ is a median of Δ ABD

, m (∠ BDC) = 30°

BC = AF = 6 cm

First: Find the length of $\overline{BD}$

Second: Prove that m (∠ BAD) = 90°



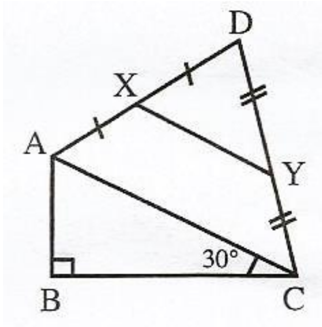
(4) In the opposite figure:

m (∠ ABC) = 90°, m (∠ ACB) = 30°

, Y and X are the midpoints of $\overline{CD}$

and $\overline{AD}$ respectively

Prove that: XY = AB



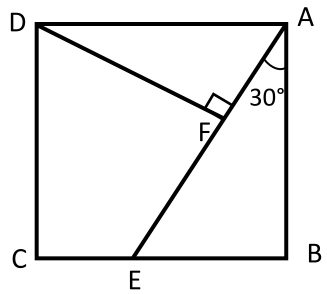
(5) In the opposite figure:

ABCD is a square, E ∈ $\overline{BC}$ such that

m (∠ BAE) = 30°, $\overline{DF} \perp \overline{AE}$

If AF = 4 cm

Calculate the area of the square.



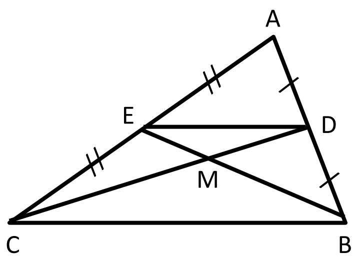
(6) In the opposite figure:

D and E are the midpoint of $\overline{AB}$ and $\overline{AC}$

respectively, BC = 10 cm, MB = 5 cm,

MC = 6 cm

Find the perimeter of Δ MDE



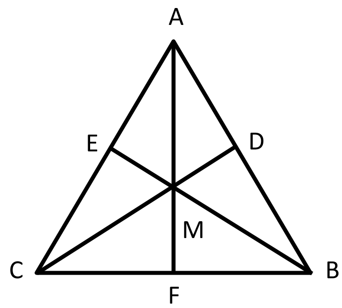
(7) In the opposite figure

If M is the point of intersection of the medians

of Δ ABC where BE = 6 cm , CD = 9 cm

and BF = 3.5 cm

Find the perimeter of Δ MBC

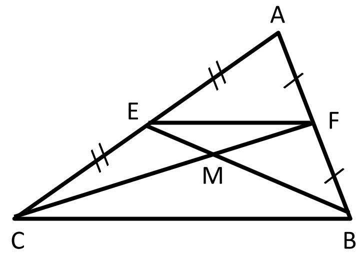


(8) In the opposite figure:

F and E are the midpoint of $\overline{AB}$ and $\overline{AC}$

in Δ ABC where BM = 5 cm , CM = 6 cm

Find the perimeter of Δ MEF

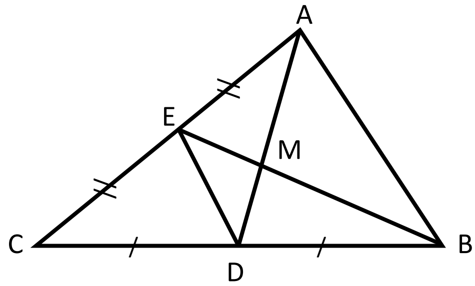


(9) In the opposite figure:

Δ ABC in which ME = 2 cm , MD = 3 cm ,

DE = 4 cm

Find the perimeter of Δ MAB

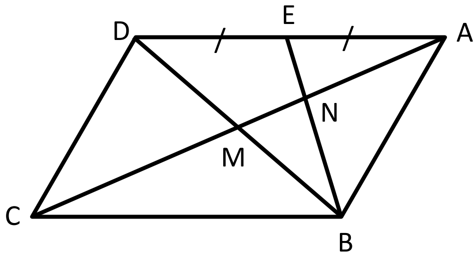


(10) In the opposite figure:

ABCD is a parallelogram, its diagonals intersect at M, E is the midpoint of $\overline{AD}$ and

$\overline{BE} \cap \overline{AC} = \{N\}$

Prove that: AN = $\frac{1}{3}$ AC

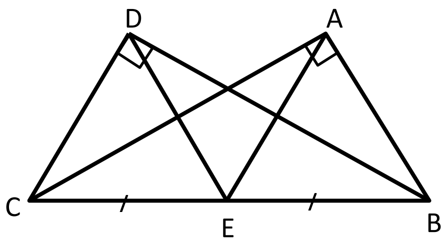


(11) In the opposite figure:

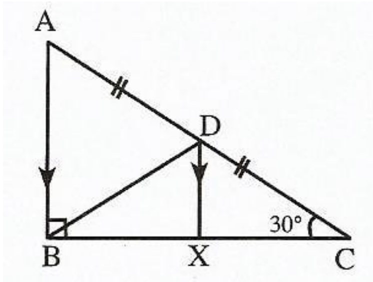
m ($\angle$ BAC) = m ($\angle$ BDC) = 90°,

E is the midpoint of $\overline{BC}$

Prove that: AE = DE



(12) In the opposite figure:

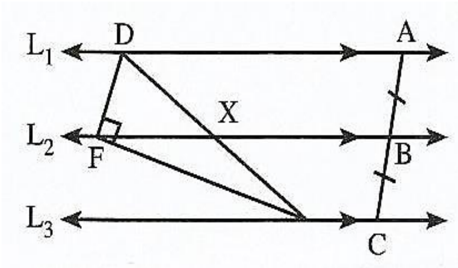


m ($\angle$ ABC) = 90° , m ($\angle$ C) = 30°

D is the midpoint of $\overline{AC}$, $\overline{DX}$ // $\overline{AB}$, AC = 12 cm

Find the length of each of BD , BA , DX

(13) In the opposite figure:

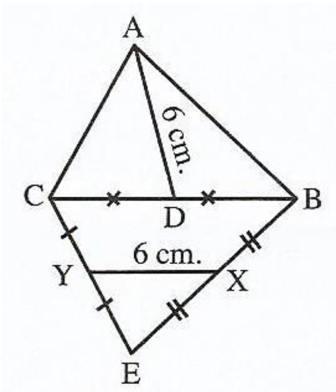


L_1 // L_2 // L_3 , AB = BC and

m ($\angle$ DFE) = 90°

Prove that: FX = $\frac{1}{2}$ DE

(14) In the opposite figure:



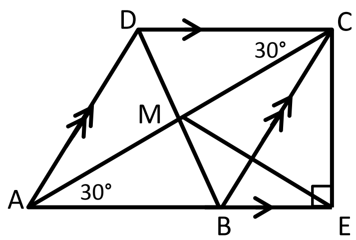
$\overline{AD}$ is a median of Δ ABC, X and Y

are the midpoint of $\overline{BE}$ and $\overline{CE}$ respectively

AD = XY = 6 cm

Prove that: m ($\angle$ BAC) = 90°

(15) In the opposite figure:



ABCD is a parallelogram M is the point of

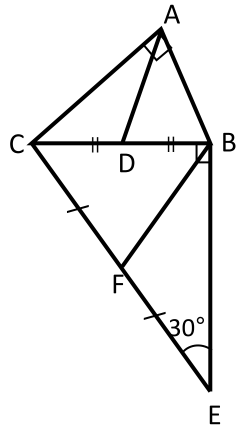
intersection of its diagonals $\overline{CE} \perp \overrightarrow{BE}$ such that

$\overline{CE} \cap \overrightarrow{BE}$ = {E} , m ($\angle$ DCA) = 30°

and AC = 18 cm

Prove that: Δ CEM is equilateral and find its perimeter.

(16) In the opposite figure:



m (∠ BAC) = m (∠ CBE) = 90°

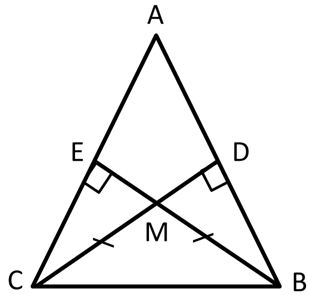
m (∠ BEC) = 30° ,

D and F are the midpoints of $\overline{BC}$

and $\overline{CE}$ respectively

Prove that: AD = $\frac{1}{2}$ BF

(17) In the opposite figure:

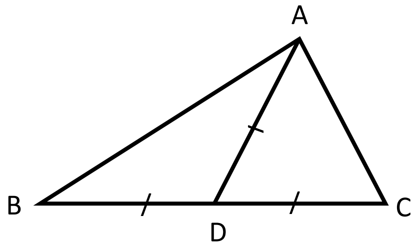


AD = AE , m (∠ ADC) = m (∠ AEB) = 90°

Prove that:

m (∠ ABC) = m (∠ ACB)

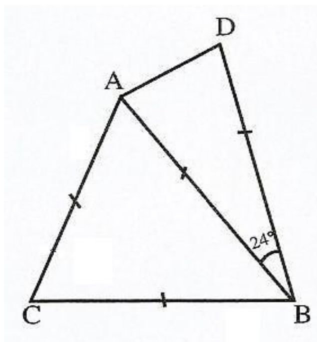
(18) In the opposite figure:



DA = DB = DC

Prove that: m (∠ BAC) = 90°

(19) In the opposite figure:



ACBD is a quadrilateral in which

AB = BC = CA = BD

, m (∠ ABD) = 24°

Find: m (∠ CAD)

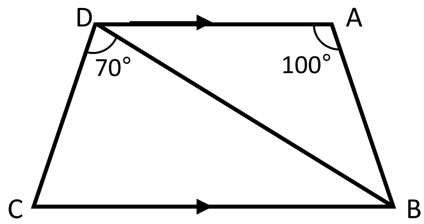
(20) In the opposite figure:

$\overline{AD} \parallel \overline{BC}$, m ($\angle$ BAD) = 100°

, m ($\angle$ BDC) = 70°

and BD = BC

Prove that: Δ ABD is isosceles

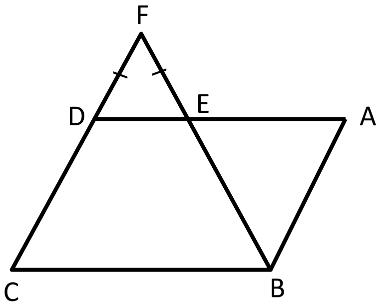


(21) In the opposite figure:

ABCD is parallelogram , E $\in \overline{AD}$

$\overrightarrow{BE} \cap \overrightarrow{CD}$ = {F} such that EF = DF

Prove that: Δ BAE is isosceles.

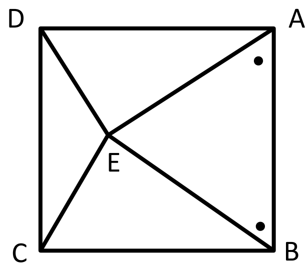


(22) In the opposite figure:

ABCD is a square and E is a point inside it

such that m ($\angle$ EAB) = m ($\angle$ EBA)

Prove that: Δ ECD is isosceles.



Model Answers

First: Complete the following:

1) a) median b) one point c) 1 : 2
d) concurrence e) $\frac{1}{2}$, 2 , $\frac{2}{3}$

2) a) 1 b) 4.5 c) 4

3) a) 6 cm b) 3 c) 3.6

4) a) $\frac{1}{2}$ the length of the hypotenuse
b) the angle at this vertex is right
c) equals half the length of hypotenuse

5) a) 4 b) 3 c) 3.5

6) a) 6 cm b) 4 cm c) 3 cm d) 9 cm

7) a) congruent b) 60°
c) congruent and the triangle is isosceles
d) the triangle is equilateral
e) an equilateral triangle
f) 60°

8) a) 45° b) 35° c) 50° d) 50° e) 45°

9) a) 50° b) 80° c) 40°

10) 42° , 48° , 62° , 115°

Second: Choose the correct answer from those given:

1) $\frac{3}{2}$ AM	2) 2 : 1	3) 4 cm	4) 3 cm
5) 120°	6) 65°	7) 100°	8) congruent
9) $\perp$	10) $\equiv$	11) rhombus	12) $\perp$

Third:

(1) Proof: $\because$ In Δ ABC

m ($\angle$ C) = 30°, m ($\angle$ ABC) = 90°, D is the midpoint of $\overline{AC}$

$\therefore$ $\overline{BD}$ is a median

$\therefore$ BD = $\frac{1}{2}$ AC (1)

$\therefore$ AB = $\frac{1}{2}$ AC (2)

$\therefore$ AB = BD = AD

$\therefore$ Δ ABD is equilateral

(2) Proof: $\because$ In Δ DEF

X is midpoint of $\overline{EF}$

$\therefore$ $\overline{DX}$ is a median, XZ = 2.5

$\therefore$ DZ = 2 ZX = 5 cm (1)

, Y is midpoint of $\overline{FD}$

$\therefore$ $\overline{EY}$ is median

EY = $\frac{1}{2}$ DF = 6 cm

EZ = $\frac{2}{3}$ EY = $\frac{2 \times 6}{3}$ = 4 cm (2)

$\because$ m ($\angle$ F) = 30°

$\therefore$ DE = $\frac{1}{2}$ FD = 6 cm (3)

P. of Δ DEZ = 6 + 4 + 5 = 15 cm

(3) Proof: ∵ In △ DCB

DC = 6 cm, m (∠ BDC) = 30°

∴ BC = $\frac{1}{2}$ DB

∴ DB = 2 × 6 = 12 cm

∵ In △ ABD

F is midpoint of $\overline{DB}$

, AF = $\frac{1}{2}$ BD

∴ m ∠ BAD = 90°

(4) Proof: ∵ In △ ACD

X, Y are midpoint of $\overline{AD}$, $\overline{DC}$

∴ XY = $\frac{1}{2}$ AC (1)

∵ In △ ABC , m (∠ C) = 30° , m (∠ B) = 90°

∴ AB = $\frac{1}{2}$ AC (2)

From (1) , (2)

XY = AB

(5) Proof: ∵ ABCD is a square

∴ m (∠ A) = 90°

∴ m (∠ DAE) = 90° - 30° = 60°

∵ In △ AFD

m (∠ ADF) = 180° - [90° + 60°] = 30°

∵ AF = 4 cm

∴ AD = 2 AF = 8 cm

∴ The area of the square = 8 × 8 = 64 cm^2

(6) Proof: $\because$ In $\triangle$ ABC

E , D are midpoints of $\overline{AC}$, $\overline{AB}$

$\therefore$ ED = $\frac{1}{2}$ CB

ED = $\frac{1}{2}$ × 10 = 5 cm (1)

$\because$ $\overline{CD}$ is median, $\overline{BE}$ is median, CM = 6 cm

$\therefore$ MD = $\frac{1}{2}$ CM = 3 cm (2)

$\because$ MB = 5 cm

$\therefore$ ME = $\frac{1}{2}$ MB = $\frac{1}{2}$ × 5 = 2.5 cm (3)

The perimeter of $\triangle$ MDE = 5 + 3 + 2.5 = 6.5 cm

(7) Proof: $\because$ M is the point of intersection of the medians of $\triangle$ ABC

$\therefore$ F is midpoint of $\overline{BC}$, FB = 3.5 cm

$\therefore$ CB = 7 cm (1)

$\because$ BE = 6 cm

$\therefore$ BM = $\frac{2}{3}$ × 6 = 4 cm (2)

$\because$ CD = 9 cm

$\therefore$ CM = $\frac{2}{3}$ × 9 = 6 cm (3)

The perimeter of $\triangle$ MBC = 7 + 4 + 6 = 17 cm

(8) Proof: $\because$ In $\triangle$ ABC

E , F are the midpoint of $\overline{AB}$ and $\overline{AC}$

$\therefore$ EF = $\frac{1}{2}$ BC = 5 cm (1)

$\because$ BM = 5 cm

$\therefore$ ME = $\frac{1}{2}$ MB = 2.5 cm (2)

$\because$ CM = 6 cm

$\therefore$ MF = $\frac{1}{2}$MC = 3 cm (3)

The perimeter of $\triangle$ MEF = 5 + 2.5 + 3 = 10.5 cm

(9) Proof: $\because$ In $\triangle$ ABC

E , D are the midpoint of $\overline{CA}$ and $\overline{CB}$

$\therefore$ ED = $\frac{1}{2}$ AB , ED = 4 cm

$\therefore$ AB = 2 × 4 = 8 cm (1)

$\because$ MD = 3 cm

$\therefore$ AM = 2 × 3 = 6 cm (2)

$\because$ ME = 2 cm

$\therefore$ MB = 2 × 2 = 4 cm (3)

The perimeter of $\triangle$ MAB = 8 + 6 + 4 = 18 cm

(10) Proof: ∵ ABCD is parallelogram

∴ AM = MC

∴ AM = $\frac{1}{2}$ AC (1)

, ∴ MB = MD

∵ E is midpoint of $\overline{AD}$

∴ $\overline{BE}$ is median

, N is the point of intersection of medians, MD = MB

∴ $\overline{AM}$ is median

∴ AN = $\frac{2}{3}$ AM (2)

From (1) in (2)

AN = $\frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}}$ AC

AN = $\frac{1}{3}$ AC

(11) Proof: ∵ In Δ ABC

m (∠ A) = 90° , CE = EB

∴ $\overline{AE}$ is median

AE = $\frac{1}{2}$ BC (1)

∵ In Δ CBD

m (∠ D) = 90°

DE = $\frac{1}{2}$ BC (2)

From (1) , (2)

AE = DE

(12) Proof: $\because$ m ($\angle$ B) = 90° , m ($\angle$ C) = 30°

D is midpoint of $\overline{AC}$

$\therefore$ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 12 = 6 cm

$\therefore$ AB = $\frac{1}{2}$ AC = 6 cm

$\because$ $\overline{DX}$ // $\overline{AB}$, D is midpoint of $\overline{AC}$

$\therefore$ XD = $\frac{1}{2}$ AB

$\therefore$ DX = $\frac{1}{2}$ × 6 = 3 cm

(13) Proof: $\because$ L_1 // L_2 // L_3 , AB = BC

$\therefore$ DX = XE

$\because$ In Δ DFE , m ($\angle$ F) = 90°

, X is midpoint of $\overline{DE}$

$\therefore$ FX = $\frac{1}{2}$ DE

(14) In Δ CBE

X and Y are the midpoints of $\overline{BE}$ and $\overline{CE}$

$\therefore$ XY = $\frac{1}{2}$ CB

CB = 2 × 6 = 12 cm

$\because$ In Δ ABC

, $\overline{AD}$ is median

$\because$ AD = $\frac{1}{2}$ CB

$\therefore$ m ($\angle$ BAC) = 90°

(15) $\because$ ABCD is parallelogram

, M is the point of intersection

, AC = 18 cm

$\therefore$ MC = 9 cm (1)

$\because$ $\overline{DC}$ // $\overline{AE}$

$\therefore$ m ($\angle$ DCA) = m ($\angle$ CAE) = 30°

$\because$ In $\triangle$ AEC

m ($\angle$ E) = 90°, m ($\angle$ CAE) = 30°

$\therefore$ $\overline{ME}$ is median

ME = $\frac{1}{2}$ AC = 9 cm (2)

$\because$ m ($\angle$ CAE) = 30°

$\therefore$ EC = $\frac{1}{2}$ AC = 9 cm (3)

$\therefore$ $\triangle$ CEM is equilateral

The perimeter of $\triangle$ CEM = 9 + 9 + 9 = 27 cm

(16) $\because$ In $\triangle$ ABC

$\because$ m ($\angle$ A) = 90° , DC = DB

$\therefore$ $\overline{AD}$ is median

AD = $\frac{1}{2}$ CB (1)

$\because$ In $\triangle$ CBE

FC = FE

$\therefore$ $\overline{FB}$ is median

FB = $\frac{1}{2}$ CE (2)

, m (∠ E) = 30°

∴ CB = $\frac{1}{2}$ CE (3)

From (2) and (3)

∴ FB = CB

∴ AD = $\frac{1}{2}$ FB

(17) Proof: ∵ In ΔΔ AEB, ADC

AD = AE
DC = EB
m (∠ ADC) = m (∠ AEB) = 90°

∴ Δ AEB ≡ Δ ADC

∴ m (∠ ABE) = m (∠ ACD) (1)

∵ In Δ MBC , MB = MC

∴ m (MBC) = m (MCB) (2)

From (1) , (2)

m (∠ ACB) = m (∠ ACB)

(18) Proof:

∵ In Δ ABC

AD = DB

∴ m (∠ DAB) = m (∠ DBA)

∵ ∠ ADC is exterior angle of Δ ADB

∴ (∠ ADC) = m (∠ DAB) + m (∠ DBA) (1)

∵ DA = DC

∴ m (∠ DAC) = m (∠ DCA)

$\because$ $\angle$ ADB is exterior angle of $\triangle$ ADC

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DAC) + m ($\angle$ DCA) (2)

$\because$ m ($\angle$ ADC) + m ($\angle$ ADB) = 180°

$\therefore$ m ($\angle$ CAD) + m ($\angle$ DAB) = m ($\angle$ BAC) = 90°

(19) Proof:

$\because$ In $\triangle$ ABC

AB = BC = AC

$\therefore$ m ($\angle$ CAB) = 60°

$\because$ In $\triangle$ ADB

AB = BD

$\therefore$ m ($\angle$ BAD) = $\frac{180°-24°}{2} = 78°$

$\therefore$ m ($\angle$ CAD) = 78° + 60° = 138°

(20) Proof:

$\because$ In $\triangle$ DBC

DB = BC

$\therefore$ m ($\angle$ BDC) = m ($\angle$ BCD) = 70°

$\therefore$ m ($\angle$ DBC) = 180° - [70° + 70°] = 40°

$\because$ $\overline{AD}$ // $\overline{BC}$

$\therefore$ m ($\angle$ ADB) = m ($\angle$ DBC) = 40°

$\because$ In $\triangle$ ABD

$\therefore$ m ($\angle$ ABD) = 180° - [40° + 100°] = 40°

$\therefore$ AB = AD

$\therefore$ $\triangle$ ABD is isosceles

(21) Proof:

$\because$ ABCD is parallelogram

$\therefore \overline{FC} \parallel \overline{AB}$

$\therefore$ m ($\angle$ FDE) = m ($\angle$ A)

$\because$ FD = FE

$\therefore$ m ($\angle$ FDE) = m ($\angle$ FED)

$\because$ m ($\angle$ FED) = m ($\angle$ AEB) (V.O.A)

$\therefore$ m ($\angle$ A) = m ($\angle$ AEB)

$\therefore$ AB = BE

$\therefore$ $\triangle$ BAF is isosceles

(22) Proof:

$\because$ ABCD is a square

$\therefore$ AD = BC (1)

, m ($\angle$ A) = m ($\angle$ B) = 90°

$\because$ m ($\angle$ EAB) = m ($\angle$ EBA)

$\therefore$ m ($\angle$ DAE) = m ($\angle$ EBC) (2)

$\because$ AE = BE (3)

From (1) , (2) , (3)

$\triangle$ AED $\equiv$ $\triangle$ BEC

$\therefore$ ED = EC

$\therefore$ $\triangle$ ECD is isosceles